Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

5702025710

FURTHER MATHEMATICS **9231/13**

Paper 1 Further Pure Mathematics 1 **October/November 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

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1 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$, where a and b are positive constants.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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The unit square in the x-y plane is transformed by $\mathbf{M}$ onto parallelogram $OPQR$.

(b) Find, in terms of a and b, the matrix which transforms parallelogram $OPQR$ onto the unit square. [2]

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It is given that the area of $OPQR$ is $2\,\text{cm}^2$ and that the line $x + 3y = 0$ is invariant under the transformation represented by $\mathbf{M}$.

(c) Find the values of a and b. [5]

2 **(a)** Use standard results from the List of Formulae (MF19) to show that

$$\sum_{r=1}^{n}(7r+1)(7r+8) = an^3 + bn^2 + cn,$$

where a, b and c are constants to be determined. [3]

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(b) Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(7r+1)(7r+8)}$ in terms of n. [4]

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(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(7r+1)(7r+8)}$. [1]

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3 The cubic equation $x^3 + cx + 1 = 0$, where c is a constant, has roots α, β, γ.

(a) Find a cubic equation whose roots are $\alpha^3, \beta^3, \gamma^3$. [3]

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(b) Show that $\alpha^6 + \beta^6 + \gamma^6 = 3 - 2c^3$. [3]

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(c) Find the real value of c for which the matrix $\begin{pmatrix} 1 & \alpha^3 & \beta^3 \\ \alpha^3 & 1 & \gamma^3 \\ \beta^3 & \gamma^3 & 1 \end{pmatrix}$ is singular. [5]

4 The points A, B, C have position vectors

$$-\mathbf{i}+\mathbf{j}+2\mathbf{k}, \quad -2\mathbf{i}-\mathbf{j}, \quad 2\mathbf{i}+2\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

(b) Find the perpendicular distance from O to the plane ABC. [2]

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(c) Find the acute angle between the planes OAB and ABC. [4]

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5 Prove by mathematical induction that, for every positive integer n,

$$\frac{\mathrm{d}^{2n-1}}{\mathrm{d}x^{2n-1}}(x\sin x) = (-1)^{n-1}\big(x\cos x + (2n-1)\sin x\big).$$ [7]

6 The curve C has equation $y = \dfrac{x^2 + x - 1}{x - 1}$.

(a) Find the equations of the asymptotes of C. [3]

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(b) Show that there is no point on C for which $1 < y < 5$. [4]

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(c) Find the coordinates of the intersections of C with the axes, and sketch C. [3]

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(d) Sketch the curve with equation $y = \left|\dfrac{x^2+x-1}{x-1}\right|$. [2]

7 **(a)** Show that the curve with Cartesian equation

$$(x^2 + y^2)^{\frac{5}{2}} = 4xy(x^2 - y^2)$$

has polar equation $r = \sin 4\theta$. [4]

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The curve C has polar equation $r = \sin 4\theta$, for $0 \leqslant \theta \leqslant \frac{1}{4}\pi$.

(b) Sketch C and state the equation of the line of symmetry. [3]

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(c) Find the exact value of the area of the region enclosed by C. [4]

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(d) Using the identity $\sin 4\theta \equiv 4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$, find the maximum distance of C from the line $\theta = \frac{1}{2}\pi$. Give your answer correct to 2 decimal places. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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